The magnetic and electrical deflectability of the Bequerel rays and the apparent mass of the electrons

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1) The question of whether the "mass" of electrons calculated from experiments on cathode rays or from the Zeemann effect is "real" or "apparent" mass has been discussed many times recently, but experimental investigations in this direction have not yet appeared. As is well known, investigations into Becquerel rays have shown that they can be deflected magnetically and electrically, and a measurement, albeit still rather crude, yielded values for both ϵ/μ (ϵ - charge, μ -mass) and for the velocity v, which were close to those found with cathode rays. It was all the more striking that the Becquerel rays differed so much from the cathode rays in quantitative terms. The magnetic deflectability of the former is much less, their ability to penetrate solid bodies is much greater. Since previous investigations of cathode rays have shown that the deflectability and the penetrating power increase with increasing speed, the assumption was obvious from the outset that the Bequerel rays differed from the cathode rays in that they had a significantly greater speed.

While the speed of the cathode rays was about 1/5 to 1/3 of the speed of light, one had to expect speeds of the Becquerel rays that deviated only slightly from the speed of light. Exceeding the speed of light, at least for a stretch of track that is large compared to the dimensions of the "electrons" (so the beam particles are called, in accordance with what is now fairly general usage) is impossible, because with such a movement energy is emitted until the speed has dropped back to the value of the speed of light.

2) The purpose of the experimental investigation described below is to determine the velocity and the ratio ϵ/μ for Bequerel rays as precisely as possible and at the same time to obtain information about the ratio of "real" and "apparent" mass from the degree of dependence obtained between ϵ/μ and v. The investigation was carried out in the Göttingen Physical Institute with the kind support of the Society of Sciences. Apart from this, I also owe thanks to Dr. Giesel in Braunschweig, who in the most courteous manner provided me with the necessary quantity of his most effective active preparation, as well as Prof. des Coudres for lending me the high-voltage battery belonging to the institute's electrotechnical department.

3) I already reported on the method used some time ago. The inhomogeneity of the Bequerel rays, which until now has mostly been felt to be bad, as a result of which a sharp bundle of rays appears to be drawn apart when deflected into a spectrum, was rendered harmless here and actually turned into an advantage by an arrangement analogous to Kundt's method of crossed spectra: By

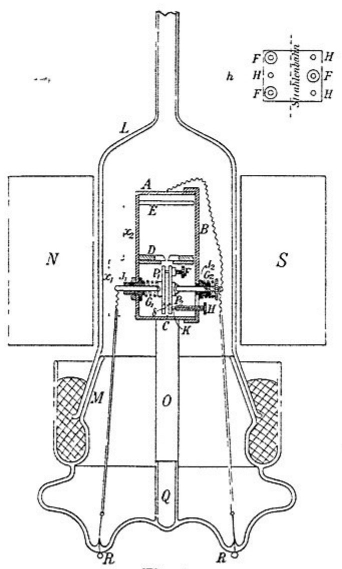

Fig. 1.

using the smallest possible granule of active substance as the source of the radiation and a fine hole as the diaphragm, a narrow bundle of rays was screened off, which appeared as a point on a photographic plate perpendicular to the direction of the radiation. Magnetic deflection turned the image into a straight line; Simultaneous electrical deflection in a direction perpendicular to the magnetic field resulted in a curve as an image, each point of which corresponded to a specific v and a specific ϵ/μ. In this way, a whole series of observations were obtained on a single plate, from which the dependence between ϵ/μ and v could be read directly.

4) Apparatus: Since the rays make the air conductive, the apparatus had to be evacuated in order to achieve a homogeneous electric field; this at the same time prevented the absorption and diffusion of the rays in the gas. Since the electromagnet required to generate the magnetic field could not be easily brought into the vacuum, and since the intensity would have been weakened too much by too large a beam path, the dimensions of the apparatus had to be chosen as small as possible. Fig. shows the apparatus in about 1/2 the natural size:

A brass box A of about $2 \times 3 \times 4.5$ cm side length is fixed by means of the tube O on the glass stem Q of a vacuum vessel L, which consists of 2 parts connected by a mercury joint M. On the bottom of the box, at C, is a granule of radium bromide about 1 mm long (longitudinally perpendicular to the plane of the drawing) and 0.3 mm thick. From the rays emanating from this, a bundle of about 1/2 mm in diameter is blocked out by the diaphragm D, which, when not blocked out, produces a small spot on the photographic plate E, which is wrapped in aluminum foil. The diaphragm consists of platinum in its middle part and lead on the sides.

An electromagnet was used to generate the magnetic field, the poles N and S of which are indicated in the figure in cross section. Since the exposure of the plate lasted 3 to 4 days, the winding was dimensioned in such a way that the magnet could be connected to the municipal light line (220 volts) with a few light bulbs connected in series. However, because of the not inconsiderable voltage fluctuations, an exact measurement of the magnetic field was impossible when using this current source; the light line was therefore only used in the preliminary tests, while the current from a 70-volt collector battery was used for the definitive measurement. Since the battery has a capacity of about 100 Ampère hours, the excitation current, which was only about 0.3 Ampère, remained completely constant during the 4-day exposure period, as determined by continuous monitoring.

The electric field was generated between two rectangular ground brass plates P_1P_2. Three screws F (only one shown in the figure) with an insulating ivory tip were used to regulate the distance between the plates. One of the plates P_1 was hinged to a handle that could be moved in the hard rubber bushing J_1 and was pressed against the tips of the screws F by the spiral spring G_1. The second, P_2, was similarly attached to the lid B of the box; an externally attached spiral spring G_2 pressed the plate against 3 screws H, also provided with ivory points, by which the position of the whole plate system relative to the apparatus was regulated. To generate the electric field, which had to remain completely constant for 4 days, I had a high-voltage battery of 2000 volts at my disposal. Through preliminary tests I convinced myself that this voltage was still too weak to measure the deflection of the fastest rays that could still be clearly observed. Following a friendly suggestion from Prof. Wiechert, I therefore constructed a potential multiplier, by means of which a voltage several times higher could be generated completely constantly for any length of time. The apparatus consisted of a rotating switch driven by an electric motor, through which the following switching operations were carried out alternately: 1) 4 (or more, if necessary) Leyden bottles are charged by the battery connected in parallel; 2) the bottles are connected in series, i.e. the voltage is quadrupled; 3) the bottles are connected to a 5th bottle which has thus been gradually charged to 4 times the potential of the battery used, repeating the process over and over again. It was enough to rotate the contact roller twice per second to achieve an absolutely steady setting of the electrometer connected to the 5th bottle. The latter was connected directly to one of the two plates P, while the other and the housing were derived to earth. The supply was made through the melted-in platinum wires RR.

An automatic pump from Sprengel's system was used for evacuation, which was connected to the apparatus by a wide glass spring. In addition to the drying vessel on the pump, there were two small vessels containing P_2O_5 (not shown in the figure) inside the apparatus.

5) Carrying out the tests: The screws F were first adjusted so that there was a distance between the plates of about 0.15 cm (the exact measurement was made at the end of the test). The photographic plate was placed in its 0.0002 cm thick aluminum foil wrapper and pressed against the projections supporting it by inserting some folded tinfoil into the space between the box and the plate above the plate. Then the lid was put on, the connection to the supply lines was made and the apparatus was brought from below into the upper part of the vacuum vessel, which was already between the magnetic poles. After about 1/2 hour of pumping, the vacuum was high enough to withstand an electrical potential difference of about 7000 volts; However, since a lot of occluded gases were always escaping at the beginning and an absolute tightness of the apparatus could not be achieved, the pump remained in operation continuously for the first 15-20 hours; later it was only pumped continuously, albeit slowly, at night; during the day pumping twice or three times for about 10 minutes is sufficient. Nevertheless, it often happened during operation that a discharge went through the apparatus; since there was always a water resistance between the collecting flask and the apparatus, only a small fraction of the charge in the flask discharged, and in the course of about 2 seconds the initial potential was restored. The light from the discharges was rendered harmless by the aluminum covering of the plate. With the good insulation of all parts, the potential difference of the plates was completely independent of fluctuations in the number of revolutions of the rotating converter. Along with the electric field, the magnetic field was also created. The constancy of the excitation current was checked from time to time by means of a torsion galvanometer; Initially, the gradual warming and associated increase in resistance of the magnet winding made frequent readjustments necessary. After the lapse of a few hours, however, the current became quite steady.

After 1 1/2-2 days the direction of the electric field was reversed and exposed again for the same time. In this way, two curve branches were obtained which were situated symmetrically to the magnetic deflection direction, so that half the distance between two corresponding curve points corresponded to the electric deflection. The plates were developed (Schleussner's moment plates) with very dilute (1:50) Rodinal developer with a lot of potassium bromide and lasted about 1/2 hour. The use of very dilute developer was necessary because the Becquerel rays, especially the weakest deflectable ones, are only slightly absorbed and therefore act uniformly through most of the layer, so the developer must have time to diffuse into the deeper parts of the layer without already fogging the surface. The images obtained, though fairly faint, were clear enough to make a measurement of the distances to about 1/200 cm (see below). Intensification with sublimate and ammonia increased the contrasts, but the settings did not become more precise as a result.

6) Measurements:

a) Dimensions of the apparatus: For the subsequent theoretical calculations it was necessary to know the following quantities: the distance x_1 from the radiation source to the diaphragm, x_2 from the diaphragm to the plate, the height h and the distance δ between the condenser plates; x_1, x_2 and h were determined using a compass and ruler. To measure δ, the cover B with the plate P_2 on it was placed horizontally on the stage of a microscope and a plane-parallel glass plate was placed on the ivory tips of the screws F. Then, by moving the micrometer, the microscope was adjusted first to the underside of the glass plate, then to P_2, and the distance was determined by multiplying the number of revolutions of the screw by the previously determined pitch of the screw.

b) Potential measurement: The potential of the battery was read on a Braun electrometer showing 0-3000 volts, the scale of which had been calibrated shortly beforehand by measuring the individual groups of the high-voltage battery using a Siemens precision voltmeter and then connecting them in series to the charging the electrometer. Multiplying the battery voltage determined in this way by 4 (since 4 Leydnen bottles were used) gave the apparatus voltage generated by the multiplier. To compare the multiplier, the latter was also read on a Braun electrometer showing up to 10,000 volts, with deviations of 50 to 100 volts compared to the calculated voltage, which are probably due to calibration errors of the second electrometer. Of course, only the information from the first calibrated electrometer came into consideration for the calculation of the results.

c) Measurement of the magnetic field: A short piece of hard rubber tubing 1.7 cm in diameter was wrapped with 20 windings of insulated copper wire and suspended from a 40 cm long brass wire with a diameter of 0.04 mm, which also served as the power supply. The derivation was effected by a downward cylindrical spiral of the same wire. The whole was contained in a glass tube provided with a window for observation of the mirror attached to the oscillating system, and surrounded by a conductive tinfoil covering for protection against electrostatic disturbances. The vibrating coil was placed in the part of the field to be measured and a current of appropriate strength was passed through it. The total deflection obtained by commuting the coil current gave a measure of the field intensity. The field along the path of the beam was determined at several points, with only minor deviations from the homogeneity (see below). The electromagnet was then replaced by a circulating current of precisely known dimensions, and the deflection obtained in the known field of the circulating current was determined.

Let n_0 be the deflection in the field H_0 of the electromagnet with the coil current $i_0 = E/W_0$ and n be the deflection in the field H of the circular current with the coil current $i = E/W$ so that:

$$H_0 = \frac{n_0}{n}\frac{W_0}{W}H \tag{1}$$

A suitable choice of W_0, W and H ensured that n_0 and n were not too different.

d) Measurement of the plate:

The photographic plate was mounted on the carriage of a 0.5 mm pitch dividing machine in a vertical position such that the direction of magnetic deflection was upward. A very weakly magnifying microscope was used for observation, in the focal plane of which there was a crosshair that could be moved micrometrically. Since the bundle of rays always contains a proportion of completely undeflectable rays (perhaps X-rays, which are generated by the electrons in the radium salt itself), the zero point of the coordinates was marked on the plate by itself. After the height of this zero point had been determined using the eyepiece micrometer, the reticle was raised a certain distance and then the horizontal distance (twice the electrical deflection) was determined by means

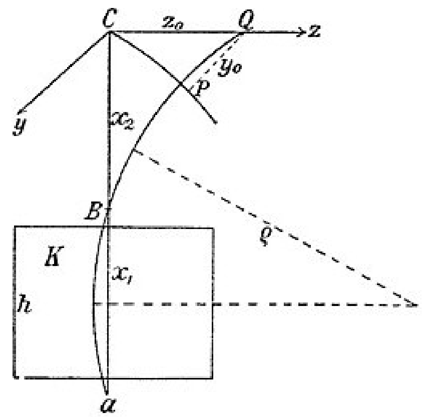

Fig. 2.

of the slide displacement. Since the curves have a certain width, they naturally fall near the zero point, i.e. partly on top of each other for the weakest deflectable rays. A measurement is therefore only possible from the point where both curves appear separately. The eyepiece micrometer was calibrated by setting it on a ruler. The values of the electrical deflection given in the results (see below) under y_0 are always the mean values of 10 measurements each, which differ from each other by a maximum of about 0.005 cm.

7) Theoretical:

Let P (Fig.) be a point on the photographic curve, Q its projection onto the direction of magnetic deflection (z), $x_0 = \overline{CQ}$ the magnetic deflection, $y_0 = \overline{PQ}$ the electrical deflection. A is the radiation source, B is the diaphragm, K is one of the capacitor plates. The X-direction is that of the undeflected bundle of rays. We consider the projection of the ray trajectory onto the $x - z$ plane. Here v_x is the projection of the velocity onto this plane, ρ is the radius of curvature of the orbital projection in the magnetic field H assumed to be constant, so that

$$\frac{v^2}{\rho} = \frac{\epsilon}{\mu} v_x H \text{ or } \frac{1}{\rho} = \frac{\epsilon}{\mu v_x} H, \tag{2}$$

where ϵ is the charge and μ is the mass of the electron.

Furthermore, if F is the electric field intensity and the condenser plates are symmetrical to A and B, then the rays from B emerge at an angle α to the $X - Z$ plane, the tangent of which is determined as follows: It is at point B

$$\frac{dy}{dt} = \frac{\epsilon}{\mu} F \frac{t}{2}, \tag{3}$$

if t is the time the particle was in the electric field. Now $dt = ds/v_x$ and $t/2 = s_1/v_x$ are understood by s_1 as the projection of half the path covered in the electric field; consequently

$$\tan\alpha = \frac{dy}{ds} = \frac{sFs_1}{\mu v_x^2} \tag{4}$$

If s_2 is the path projection from B to Q, then the electrical deflection is:

$$y_0 = \frac{sFs_1s_2}{\mu v_x^2} \tag{5}$$

If x_1 and x_2 (see 6.) are known, the approximate relationship between ρ and the magnetic deflection z_0 can easily be established:

$$\rho = \frac{z_0^2 + x_2^2 + x_1x_2}{2z_0} - \frac{x_1^2 z_0}{4z_0^2 + x_2^2 + x_1x_2} \tag{6}$$

or, since $x_1 = 2.07$ cm and $x_2 = 2$ cm:

$$\rho = \frac{z_0^2 + 8.15}{2z_0} - \frac{4.29z_0}{4z_0^2 + 8.15} \tag{7}$$

furthermore, the height of the condenser plates is $h = 1,775$ cm, i.e.

$$\begin{cases} s_1 = \rho \arcsin \frac{1.775}{2\rho} \\ s_2 = 2\rho \arcsin \frac{\sqrt{4+z_0^2}}{2\rho} \end{cases} \tag{8}$$

From 2) and 5) results:

$$v_x = \frac{Fs_1s_2}{y_0\rho H} \tag{9}$$

$$\frac{\epsilon}{\mu} = \frac{v_x}{\rho H} \tag{10}$$

Finally, it is easy to see the real orbital velocity, which is again exactly as great in B as in A:

$$v = v_x \left[1 + \frac{1}{2}\frac{y_0^2}{s_2^2}\right]; \tag{11}$$

however, given the smallness of y_0/s_2, $v_x = v$ can be set.

8) Result: Two plates proved to be suitable for the measurement, but only one of them was recorded with a completely constant electric field. I therefore only quote the results of this plate; the results of the other, less precise, deviate from the first by 3 to 5%, so are qualitatively identical. The test data are as follows:

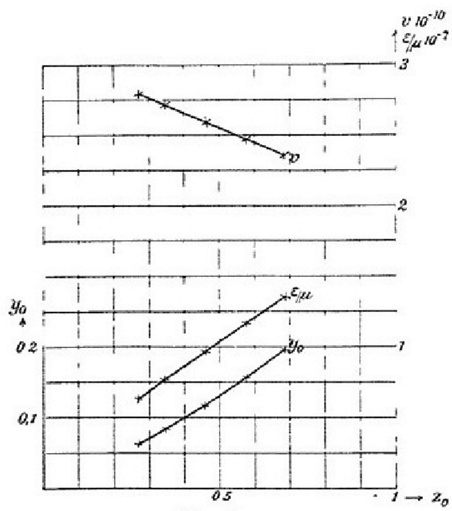

Fig. 3.

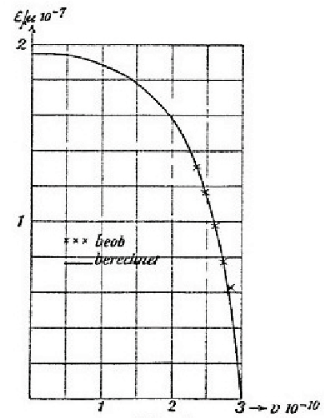

Fig. 4.

Exposure time twice 48 hours,

distance of capacitor plates $\delta = 0.1525$ cm

potential difference $\phi = 6750$ volt $= 6750 \times 10^8$C.G.S.E.

Consequently $F = \frac{6750 \times 10^8}{0.1525} = 44.3 \times 10^{11}$

Furthermore, the mean value of the magnetic field:

$$H = 299; \left[\frac{H_{max} - H_{min}}{H} \times 100 = 7.5\% \right].$$

Table 1: All numbers in absolute terms

z_0	y_0	ρ	s_1	s_2	$v_x \cdot 10^{-10}$	$\epsilon/\mu \cdot 10^{-7}$
0.271	0.0621	15.1	0.888	2.02	2.83	0.63
0.348	0.0839	11.7	0.888	2.03	2.72	0.77
0.461	0.1175	8.9	0.889	2.06	2.59	0.975
0.576	0.1565	7.1	0.889	2.09	2.48	1.17
0.688	0.198	6.0	0.890	2.13	2.36	1.31

A graphical representation of the results is given in Fig. 3 and 4. [About the “calculated” curve in Fig. 4 see below.]

As far as the accuracy of the results is concerned, the relative accuracy of the individual numbers is much greater than the absolute one, since the latter involves a much larger series of individual determinations. After all, the absolute values obtained should be safe to about 5%.

9) True and apparent mass:

One sees from the reported numbers that the speed of the fastest measurable rays is only slightly behind the speed of light. From the curve for v in Fig. 3 it appears that the velocity for the more weakly deflectable rays converges towards the speed of light. ϵ/μ varies greatly in the interval observed; as v increases, ϵ/μ decreases sharply, which would result in a not inconsiderable proportion of "apparent" mass, which the latter must increase when approaching the speed of light in order to become infinitely large when it is reached.

A rigorous formula for the field energy of rapidly moving electrons has been derived by Searle, based on the assumption that an electron is equivalent to a charged, infinitely thin spherical shell. If a is the radius of the sphere, V is the speed of light, v is the speed of the electron, e is its charge in electromagnetic terms, then the field energy (electrostatic + electromagnetic energy) is

$$W = \frac{\epsilon^2 V^2}{2a}\left[\frac{1}{\beta}\log\frac{1+\beta}{1-\beta} - 1\right] \text{ where } \beta = v/V \tag{12}$$

From this it follows for the apparent mass:

$$m = \frac{1}{v}\frac{dW}{dv} = \frac{\epsilon^2}{2a}\frac{1}{\beta^2}\left[\frac{1}{\beta}\log\frac{1+\beta}{1-\beta} + \frac{2}{1-\beta^2}\right] \tag{13}$$

or in series development:

$$m = \frac{2}{3}\frac{\epsilon^2}{a}\left[1 + \frac{3}{2}\frac{4}{5}\beta^2 + \frac{3}{2}\frac{6}{7}\beta^4 + \frac{3}{2}\frac{8}{9}\beta^6 \ldots\right] \tag{14}$$

for very small β becomes

$$m = m_0 = \frac{2}{3}\frac{\epsilon^2}{a} \tag{15}$$

so that you get:

$$\begin{aligned} \eta = \frac{m}{m_0} &= \frac{3}{4\beta^2}\left[\frac{1}{\beta}\log\frac{1-\beta}{1+\beta} + \frac{2}{1-\beta^2}\right] \\ &= 1 + \frac{3}{2}\frac{4}{5}\beta^2 + \frac{3}{2}\frac{6}{7}\beta^4 + \ldots \end{aligned} \tag{16}$$

[For β almost equal to 1, the series converges extremely slowly, so that a large number of terms have to be taken into account.]

Now let M be the true mass of the electron, m the total mass so that

$$\mu = M + m = M + m_0 \cdot \eta \tag{17}$$

and

$$\epsilon/\mu = \epsilon/(M + m_0\eta).$$

If v is known, then η is also known, so the most probable value of M/ϵ and m_0/ϵ can be calculated using the least squares method.

Leaving out the numbers in the first line of Table 7 (which are too uncertain due to the smallness of the observed deflection), the calculation yields the most probable values:

$$\left\{\begin{array}{l} M' = 0.39 \cdot 10^{-7} \\ m_0' = 0.122 \cdot 10^{-7} \end{array}\right. \tag{18}$$

hence for very slow rays

$$\epsilon/\mu_0 = \frac{1}{M' + m_0'} = 1.95 \cdot 10^7. \tag{19}$$

a value that agrees sufficiently with that found for cathode rays ($1.865 \cdot 10^7$). Table II gives a summary of the observed and according to Eq. 16), 18) and 19) calculated values:

Table 2					
$10^{-10}v$	β	η	$\epsilon/\mu \cdot 10^{-7}$		
			beob.	ber	Diff%
[2.83]	[0.945]	[12.5]	[1.59]	[1.91]	
2.72	0.907	7.41	1.30	1.29	+0.8
2.59	0.864	4.88	1.025	0.99	+3.5
2.48	0.827	3.85	0.855	0.86	-0.6
2.36	0.787	3.13	0.765	0.77	-0.6

With the exception of the first value, which is too uncertain as mentioned above, the formula represents the observations fairly well, as is particularly evident from the calculated curve for $\epsilon/\mu = 1/\mu$ shown in Fig. 4. The ratio of apparent to real mass is therefore for velocities that are small compared to the speed of light:

$$\frac{m_0}{M} = \frac{m_0'}{M'} = \frac{0.122}{0.39} = 0.313 \text{ or approximately } 1/3 \tag{20}$$

Even if the latter number is still subject to considerable uncertainty (an error of 10% in the constants governing the magnetic deflection would make the actual mass vanishingly small), one can nevertheless claim on the basis of the above results that the apparent mass of is of the same order of magnitude as the real one, and in the case of the fastest Bequerel rays it even surpasses the latter considerably.

Finally, it should be expressly pointed out that the prerequisite for the above consideration is the assumed distribution of the charge of the electron on an infinitely thin spherical shell. Since we know nothing at all about the constitution of the electron and are not a priori entitled to apply the electrostatic laws, which are to be explained with the help of the electrons, to the latter itself, it is of course just as possible that the energy relationships of the electron through other charge distributions can be represented, and that there are also such which, applied to the above observations, cause the real mass to disappear completely. It would be very desirable if the calculation of the field energy for other charge distributions, such as for a charged sphere, were carried out at once. Unfortunately, the interesting calculations by C.H. Wind only refer to slow speeds.